Blacktown Boys' High School

2024

HSC Trial Examination

Mathematics Extension 1

General Instructions	• Reading time – 10 minutes • Working time – 2 hours • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided for this paper • All diagrams are not drawn to scale • In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
Total marks: 70	**Section I – 10 marks** (pages 3 – 7) • Attempt Questions 1 – 10 • Allow about 15 minutes for this section **Section II – 60 marks** (pages 8 – 15) • Attempt Questions 11 – 14 • Allow about 1 hour and 45 minutes for this section

Assessor: X. Chirgwin

Student Name: ____

Teacher Name: ____

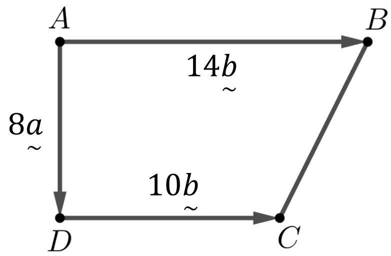
Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2024 Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

Q1. Which of the following is the correct vector of $\overrightarrow{BC}$?



A. $-8\underset{\sim}{a} - 4\underset{\sim}{b}$

B. $-8\underset{\sim}{a} + 4\underset{\sim}{b}$

C. $8\underset{\sim}{a} - 4\underset{\sim}{b}$

D. $8\underset{\sim}{a} + 4\underset{\sim}{b}$

Q2. A standard six-sided die is rolled 18 times.

Let $\hat{p}$ be the proportion of the rolls with an outcome of 3.

Which of the following expressions is the probability that at most 12 of the rolls have an outcome of 3?

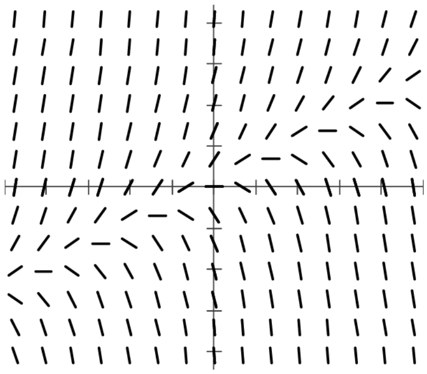
A. $P\left(\hat{p} \leq \frac{1}{6}\right)$

B. $P\left(\hat{p} \leq \frac{2}{3}\right)$

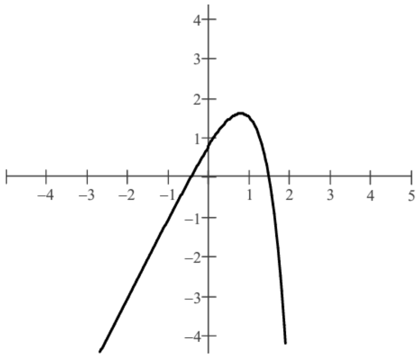
C. $P\left(\hat{p} \geq \frac{1}{6}\right)$

D. $P\left(\hat{p} \geq \frac{2}{3}\right)$

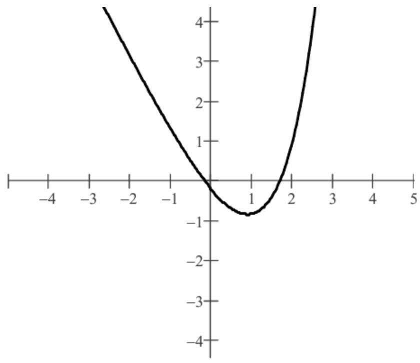
Q3. Which of the following could be the graph of the solution of the differential equation shown?



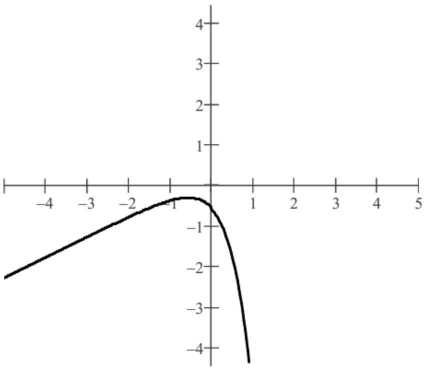
A.



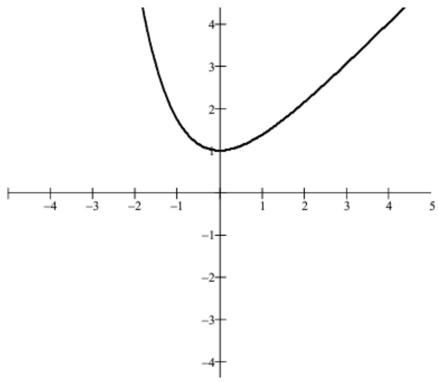
B.



C.



D.



Q4. A graph has parametric equations $x = 2\cos t$, $y = 2 - 4\sin^2 t$. What is its Cartesian equation?

A. $y = x^2 - 2$ for $-2 \leq x \leq 2$

B. $y = x^2 - 2$ for $-2 \leq x \leq 0$

C. $y = x^2 + 2$ for $-2 \leq x \leq 2$

D. $y = x^2 + 2$ for $-2 \leq x \leq 0$

Q5. If $\sin x = 0.28$ and $\frac{\pi}{2} \leq x \leq \pi$, evaluate $\tan 2x$.

A. $\frac{527}{336}$

B. $-\frac{527}{336}$

C. $\frac{336}{527}$

D. $-\frac{336}{527}$

Q6. Mohammad hits the target on average 2 out of every 3 shots in archery competitions. During a competition he has 10 shots at the target. What is the probability that Mohammad hits the target exactly 9 times?

A. $10\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^9$

B. $\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)$

C. $10\left(\frac{2}{3}\right)^9$

D. $5\left(\frac{2}{3}\right)^{10}$

Q7. What is the value of $\tan\alpha$ when the expression $4\sin x - 3\cos x$ is written in the form $5\sin(x - \alpha)$?

A. $-\frac{3}{4}$

B. $\frac{3}{4}$

C. $-\frac{4}{3}$

D. $\frac{4}{3}$

Q8. What is the range of the function $f(x) = \cos^{-1}(\tan x)$?

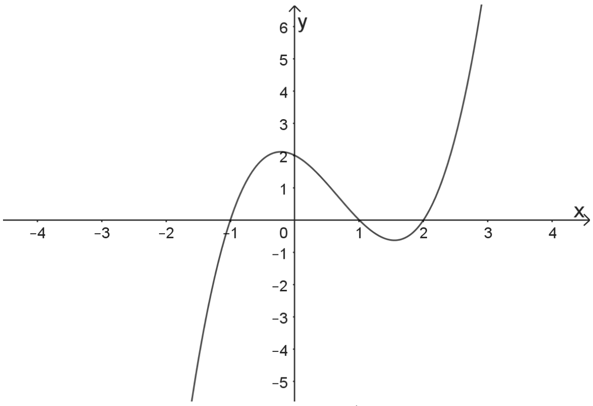
A. $[0, \pi]$

B. $(0, \pi)$

C. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

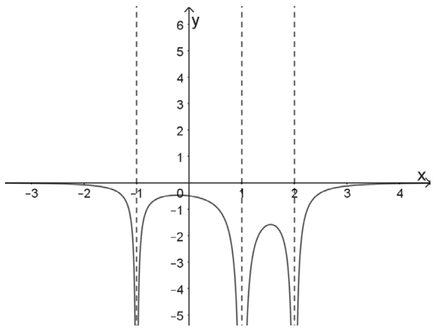
D. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Q9. The graph of the function $y = f(x)$ is below.

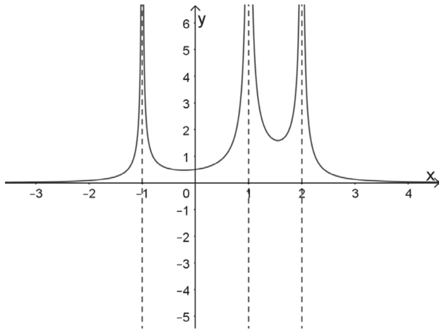


Which of the following is a graph of $y = \dfrac{-1}{f(|x|)}$?

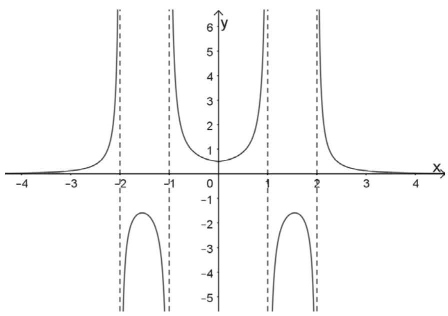
A.



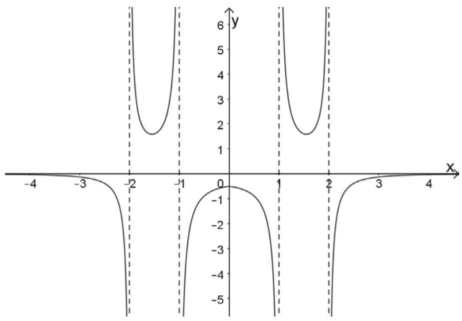
B.



C.



D.



Q10. Given that $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are two non-zero vectors, let $\underset{\sim}{c}$ be the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$. What is the projection of $12\underset{\sim}{a}$ onto $3\underset{\sim}{b}$?

A. $36\underset{\sim}{c}$

B. $12\underset{\sim}{c}$

C. $4\underset{\sim}{c}$

D. $3\underset{\sim}{c}$

End of Section I

Section II
60 Marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question on a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

(a) For the vectors $\underset{\sim}{u} = 5\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$, evaluate each of the following.

(i) $2\underset{\sim}{u} - 3\underset{\sim}{v}$ **1**

(ii) $\underset{\sim}{u} \cdot \underset{\sim}{v}$ **1**

(b) The polynomial $P(x) = 5x^3 - 2x + 20$ has roots α, β and γ. Evaluate:

(i) $\alpha\beta\gamma$ **1**

(ii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

(c) Solve $\dfrac{10x}{1+3x} \leq 3$ **3**

(d) Show that $\dfrac{d}{dx}\left(\dfrac{\sin^{-1}(3x)}{3x}\right) = \dfrac{3x - \sin^{-1}(3x)\sqrt{1-9x^2}}{3x^2\sqrt{1-9x^2}}$ **3**

Question 11 continues on next page

Question 11 (continued)

(e) A recent census found that 55% of people used public transport.

A sample of 600 randomly selected Australians was surveyed.

Let $\hat{p}$ be the sample proportion of surveyed people who were born overseas.

A normal distribution is to be used to approximate $P(\hat{p} \leq 0.575)$.

(i) Show that the variance of the random variable $\hat{p}$ is $\dfrac{33}{80000}$. **2**

(ii) Use the standard normal distribution and the information on page 15 to approximate $P(\hat{p} \leq 0.575)$, giving your answer correct to two decimal places. **2**

End of Questions 11

Question 12 (15 marks) Use the Question 12 Writing Booklet.

(a) Evaluate $\displaystyle\int_0^{\frac{1}{5}} \frac{dx}{1+25x^2}$ **2**

(b) Evaluate $\displaystyle\int_{-1}^{4} \frac{t}{\sqrt{5+t}}\,dt$ by using the substitution $t = u - 5$. **3**

(c) Samirali turns on his radio 30 times a month on average. It can receive 20 radio stations. Every time he switches on the radio, it goes to a random station. What is the probability that it will be the radio station Samirali wants to listen to upon switching on the ratio at least twice in a month? Round your answer to 4 significant figures. **3**

(d) It is known that ${}^{n}C_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$ for all integers such that $1 \le r \le n-1$. (Do NOT prove this.) **2**

Find ONE possible set of values for p and q such that

$$^{2026}C_{1924} - {}^{2024}C_{100} - {}^{2024}C_{101} = {}^{p}C_q$$

(e) The vectors $\underset{\sim}{u} = \begin{pmatrix} -5 \\ m \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 3m-4 \\ 1-6m \end{pmatrix}$ are perpendicular. **2**

What are the possible values of m?

(f) For all integers $n \ge 1$, use mathematical induction to prove that **3**

$$\frac{3}{1\times 2\times 2} + \frac{4}{2\times 3\times 2^2} + \frac{5}{3\times 4\times 2^3} + \cdots + \frac{n+2}{n(n+1)\times 2^n} = 1 - \frac{1}{(n+1)\times 2^n}$$

End of Questions 12

Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a) (i) By expanding the left-hand side, show that **1**

$$\sin(8x + 5x) + \sin(8x - 5x) = 2\sin 8x \cos 5x$$

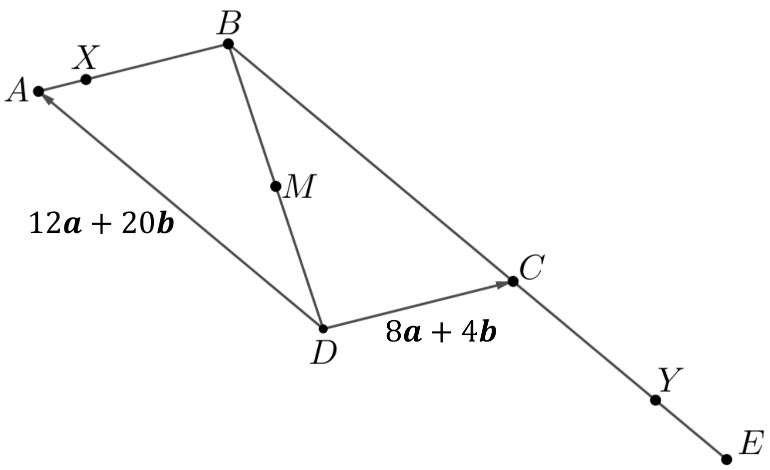
(ii) Hence find $\int \sin 8x \cos 5x \, dx$. **2**

(b) The arc of the curve $y = \frac{1}{2}(1 + \sin x)$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is rotated about the x-axis. Find the volume of the solid formed. **3**

(c) $ABCD$ is a parallelogram where $\overrightarrow{DA} = 12\boldsymbol{a} + 20\boldsymbol{b}$ and $\overrightarrow{DC} = 8\boldsymbol{a} + 4\boldsymbol{b}$.

X lies on the line $\overrightarrow{AB}$ such that $\overrightarrow{AX}:\overrightarrow{XB} = 1:3$. M is the midpoint of $\overrightarrow{DB}$.

$\overrightarrow{CE}$ is an extension of $\overrightarrow{BC}$. Y lies on the line $\overrightarrow{CE}$ such that $2\overrightarrow{CY} = -\overrightarrow{DA}$.



(i) Find $\overrightarrow{XM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. **2**

(ii) Prove that X, M and Y are collinear and find k if $\overrightarrow{XM} = k\overrightarrow{MY}$. **3**

Question 13 continues on next page

Question 13 (continued)

(d) (i) Show that $\dfrac{x+2}{4-x^2} = \dfrac{1}{2-x}$ **1**

(ii) Find the particular solution to the differential equation **3**

$$2(4-x^2)\frac{dy}{dx} = y(x+2)$$

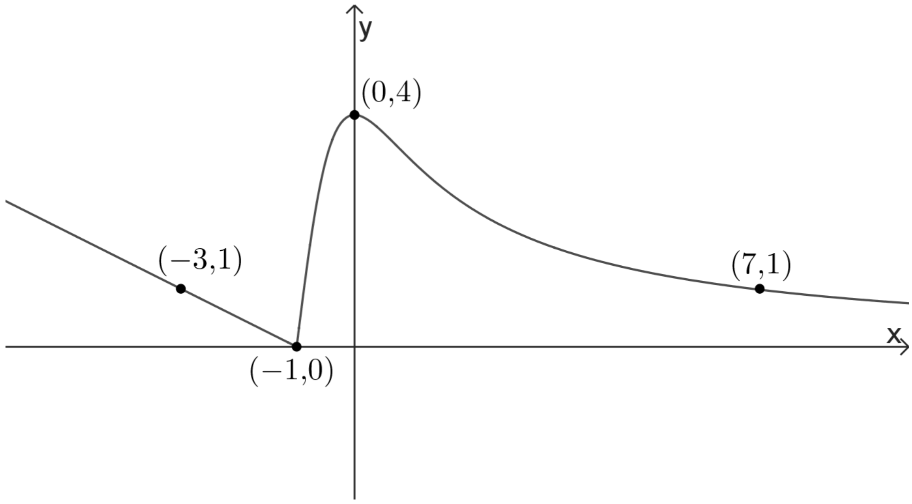
that passes through the point $(1, 2)$, give the answer in the form

$y = f(x)$.

End of Questions 13

Question 14 (15 marks) Use the Question 14 Writing Booklet.

(a) The diagram below shows the graph of the function $y = f(x)$. **3**



Sketch $y = \dfrac{1}{\sqrt{f(x)}}$

(b) The rate of change of the number of people entering an arena is modelled by $\dfrac{dN}{dt} = kN\left(1 - \dfrac{N}{5000}\right)$, where k is a constant and N is the number of people after t minutes. There are 200 people in the arena initially and the capacity of the arena is 5000 people. At a certain time, the number of people in the arena is 1000 and is increasing at the rate of 500 people per minute.

(i) Show that $k = 0.625$. **1**

(ii) Show that $\dfrac{5000}{N(5000 - N)} = \dfrac{1}{N} + \dfrac{1}{5000 - N}$ **1**

(iii) Find the number of people in the arena after 6 minutes. **4**

(iv) How long will it take for the arena to be 90% full? Round to the nearest second. **1**

Question 14 continues on next page

Question 14 (continued)

(c) Given that $\sin^{-1} x$, $\cos^{-1} x$ and $\sin^{-1}(2-x)$ have values for $0 \leq x \leq \dfrac{\pi}{2}$.

(i) Show that $\sin(\sin^{-1} x - \cos^{-1} x) = 2x^2 - 1$. **3**

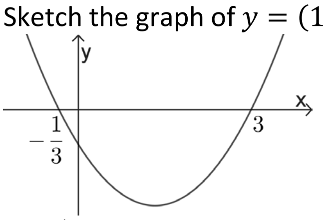
(ii) Hence solve the equation $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(2-x)$. **2**

End of Paper

2024 Mathematics Extension 1 AT4 Trial Solutions

Section 1		
Q1	**C** $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AD} + \overrightarrow{DC}$ $\overrightarrow{BC} = -14\underset{\sim}{b} + 8\underset{\sim}{a} + 10\underset{\sim}{b}$ $\overrightarrow{BC} = 8\underset{\sim}{a} - 4\underset{\sim}{b}$	1 Mark
Q2	**B** Having 12 out of 18 rolls be an outcome of 3 corresponding to a $\hat{p}$ value of $\frac{12}{18} = \frac{2}{3}$ Since "at most" that proportion is desired, then probability is $P\left(\hat{p} \le \frac{2}{3}\right)$	1 Mark
Q3	**C**	1 Mark
Q4	**A** $x = 2\cos t, \quad -2 \le x \le 2$ $x^2 = 4\cos^2 t$ $y = 2 - 4\sin^2 t$ $4\sin^2 t = 2 - y$ $\sin^2 t + \cos^2 t = 1$ $4\sin^2 t + 4\cos^2 t = 4$ $2 - y + x^2 = 4$ $y = x^2 - 2$	1 Mark
Q5	**D** $\sin x = \frac{7}{25}, \ \frac{\pi}{2} \le x \le \pi$ $\tan x = -\frac{7}{24}$ $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$ $\tan 2x = \frac{2 \times \left(-\frac{7}{24}\right)}{1 - \left(-\frac{7}{24}\right)^2}$ $\tan 2x = -\frac{336}{527}$	1 Mark
Q6	**D** ${}^{10}C_9 \left(\frac{2}{3}\right)^9 \left(\frac{1}{3}\right)^1 = 5 \times \left(\frac{2}{3}\right)^{10}$	1 Mark
Q7	**B** $5\sin(x - \alpha) = 5\sin x\cos\alpha - 5\cos x\sin\alpha$ $5\cos\alpha = 4 \ldots (1)$ $5\sin\alpha = 3 \ldots (2)$ $(2) \div (1)$ $\tan\alpha = \frac{3}{4}$	1 Mark
Q8	**A** Range is $0 \le y \le \pi$	1 Mark

Q9	**D** $y = \dfrac{-1}{f(\lvert x \rvert)}$ Take the graph where x is positive and reflect it along the y-axis. Then take the reciprocal of this graph, generating vertical asymptotes at $x = \pm 1, \pm 2$, and y-intercept becomes $\frac{1}{2}$. Then reflect the resulting graph along the x-axis, and y-intecept becomes $-\frac{1}{2}$.	1 Mark
Q10	**B** $\text{proj}_{(3\underset{\sim}{b})}\left(12\underset{\sim}{a}\right) = \dfrac{\left(12\underset{\sim}{a}\right)\cdot\left(3\underset{\sim}{b}\right)}{\left\lvert 3\underset{\sim}{b}\right\rvert^2}\left(3\underset{\sim}{b}\right)$ $\text{proj}_{(3\underset{\sim}{b})}\left(12\underset{\sim}{a}\right) = \dfrac{12\left(\underset{\sim}{a}\cdot\underset{\sim}{b}\right)}{\left\lvert\underset{\sim}{b}\right\rvert^2}\left(\underset{\sim}{b}\right)$ $\text{proj}_{(3\underset{\sim}{b})}\left(12\underset{\sim}{a}\right) = 12\text{proj}_{(\underset{\sim}{b})}\left(\underset{\sim}{a}\right)$ $\text{proj}_{(3\underset{\sim}{b})}\left(12\underset{\sim}{a}\right) = 12\underset{\sim}{c}$	1 Mark

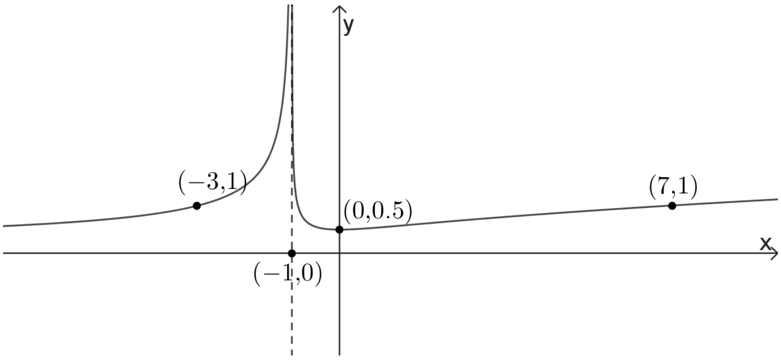
Section 2		
Q11ai	$2\underset{\sim}{u} - 3\underset{\sim}{v}$ $= 2\left(5\underset{\sim}{i} + \underset{\sim}{j}\right) - 3\left(2\underset{\sim}{i} - 3\underset{\sim}{j}\right)$ $= 10\underset{\sim}{i} + 2\underset{\sim}{j} - 6\underset{\sim}{i} + 9\underset{\sim}{j}$ $= 4\underset{\sim}{i} + 11\underset{\sim}{j}$	1 Mark Correct solution
Q11aii	$\left(5\underset{\sim}{i} + \underset{\sim}{j}\right) \cdot \left(2\underset{\sim}{i} - 3\underset{\sim}{j}\right)$ $= 5 \times 2 + 1 \times -3$ $= 7$	1 Mark Correct solution
Q11bi	$P(x) = 5x^3 - 2x + 20$ $\alpha\beta\gamma = -\frac{d}{a} = -\frac{20}{5}$ $\alpha\beta\gamma = -4$	1 Mark Correct solution
Q11bii	$\alpha^2 + \beta^2 + \gamma^2$ $= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$ $= 0 - 2 \times -\frac{2}{5}$ $= \frac{4}{5}$	2 Marks Correct solution 1 Mark Makes significant progress
Q11c	$\frac{10x}{1+3x} \leq 3, \quad x \neq -\frac{1}{3}$ $10x(1+3x) \leq 3(1+3x)^2$ $10x(1+3x) - 3(1+3x)^2 \leq 0$ $(1+3x)[10x - 3(1+3x)] \leq 0$ $(1+3x)(x-3) \leq 0$ Sketch the graph of $y = (1+3x)(x-3)$  $\therefore -\frac{1}{3} < x \leq 3$	3 Marks Correct solution 2 Marks Makes significant progress and identifies 3, $-\frac{1}{3}$ are the two key values 1 Mark Multiplies both sides by the square of denominator
Q11d	$\frac{d}{dx}\left(\frac{\sin^{-1}(3x)}{3x}\right)$ $= \frac{3x \times \frac{3}{\sqrt{1-9x^2}} - 3\sin^{-1}(3x)}{9x^2}$ $= \frac{3\left(3x - \sin^{-1}(3x)\sqrt{1-9x^2}\right)}{9x^2\sqrt{1-9x^2}}$ $= \frac{3x - \sin^{-1}(3x)\sqrt{1-9x^2}}{3x^2\sqrt{1-9x^2}}$	3 Marks Correct solution 2 Marks Make significant progress 1 Mark Differentiate $\sin^{-1}(3x)$
Q11ei	Let p be the probability of people who catch public transport $p = 0.55, \qquad q = 1 - p = 0.45$ $n = 600$ $\sigma^2 = \frac{0.55 \times 0.45}{600}$ $\sigma^2 = \frac{33}{80000}$	2 Marks Correct solution 1 Mark Obtains correct p and q values

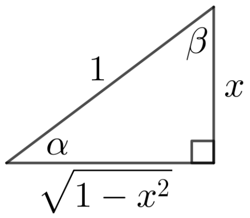
Q11eii	$P(\hat{p} \leq 0.575) = P\left(Z \leq \frac{0.575 - 0.55}{\sqrt{\frac{33}{80000}}}\right)$ $P(\hat{p} \leq 0.575) = P(Z \leq 1.2309 \ldots)$ $P(\hat{p} \leq 0.575) \approx P(Z \leq 1.23)$ $P(\hat{p} \leq 0.575) \approx 0.8907$ $P(\hat{p} \leq 0.575) = 0.89$ (2 dp)	2 Marks Correct solution 1 Mark Obtains the correct z-score
Q12a	$\int_0^{\frac{1}{5}} \frac{dx}{1+25x^2}$ $= \frac{1}{5}\int_0^{\frac{1}{5}} \frac{5dx}{1+(5x)^2}$ $= \frac{1}{5}[\tan^{-1}(5x)]_0^{\frac{1}{5}}$ $= \frac{1}{5}\left[\tan^{-1}\left(5 \times \frac{1}{5}\right) - \tan^{-1}(5 \times 0)\right]$ $= \frac{1}{5}\left[\frac{\pi}{4} - 0\right]$ $= \frac{\pi}{20}$	2 Marks Correct solution 1 Mark Correct anti-derivative
Q12b	$I = \int_{-1}^{4} \frac{t}{\sqrt{5+t}} dt$ $t = u - 5$ $dt = du$ $t = 4, u = 9$ $t = -1, u = 4$ $I = \int_4^9 \frac{u-5}{\sqrt{(5+u-5)}} du$ $I = \int_4^9 \frac{u-5}{\sqrt{u}} du$ $I = \int_4^9 \left(u^{\frac{1}{2}} - 5u^{-\frac{1}{2}}\right) du$ $I = \left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{5u^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^9$ $I = \left[\frac{2u\sqrt{u}}{3} - 10\sqrt{u}\right]_4^9$ $I = \left[\left(\frac{2 \times 9\sqrt{9}}{3} - 10\sqrt{9}\right) - \left(\frac{2 \times 4\sqrt{4}}{3} - 10\sqrt{4}\right)\right]$ $I = \left[(18 - 30) - \left(\frac{16}{3} - 20\right)\right]$ $I = \frac{8}{3}$	3 Marks Correct solution 2 Marks Obtains correct anti-derivative in terms of u 1 Mark Obtains correct integrand in terms of u
Q12c	The probability of the correct radio station is $\frac{1}{20}$ $X \sim B\left(30, \frac{1}{20}\right)$ $P(X \geq 2) = 1 - P(X \leq 1)$ $P(X \geq 2) = 1 - \big(P(X = 1) + P(X = 0)\big)$	3 Marks Correct solution 2 Marks Makes significant progress

	$P(X \geq 2) = 1 - \left({}^{30}C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{30} + {}^{30}C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^{29} \right)$ $P(X \geq 2) = 0.446457 \ldots$ $P(X \geq 2) = 0.4465$ (4 sig fig)	1 Mark Finds $P(X = 0)$ or $P(X = 1)$
Q12d	${}^{2026}C_{1924} - {}^{2024}C_{100} - {}^{2024}C_{101} = {}^{p}C_q$ ${}^{2026}C_{1924} - ({}^{2024}C_{100} + {}^{2024}C_{101}) = {}^{p}C_q$ ${}^{2026}C_{1924} - {}^{2025}C_{101} = {}^{p}C_q$ ${}^{p}C_q + {}^{2025}C_{101} = {}^{2026}C_{1924}$ ${}^{p}C_q + {}^{2025}C_{101} = {}^{2026}C_{102}$ ${}^{2025}C_{102} + {}^{2025}C_{101} = {}^{2026}C_{102}$ $\therefore p = 2025, q = 102 \text{ or } 1923$	2 Marks Correct solution 1 Mark Combines two terms
Q12e	$\begin{pmatrix} -5 \\ m \end{pmatrix} \cdot \begin{pmatrix} 3m-4 \\ 1-6m \end{pmatrix} = 0$ $-15m + 20 + m - 6m^2 = 0$ $-6m^2 - 14m + 20 = 0$ $3m^2 + 7m - 10 = 0$ $(3m+10)(m-1) = 0$ $m = -\frac{10}{3}, m = 1$	2 Marks Correct solution 1 Mark Makes significant progress
Q12f	$RTP: \frac{3}{1 \times 2 \times 2} + \frac{4}{2 \times 3 \times 2^2} + \frac{5}{3 \times 4 \times 2^3} + \cdots + \frac{n+2}{n(n+1) \times 2^n}$ $= 1 - \frac{1}{(n+1) \times 2^n}$ 1. Prove statement is true for $n = 1$ $LHS = \frac{3}{1 \times 2 \times 2} = \frac{3}{4}$ $LHS = \frac{3}{4}$ $RHS = 1 - \frac{1}{(1+1) \times 2^1}$ $RHS = 1 - \frac{1}{4}$ $RHS = \frac{3}{4}$ $\therefore$ statement is true for $n = 1$ 2. Assume statement is true for $n = k$ (k is some positive integer) $\frac{3}{1 \times 2 \times 2} + \frac{4}{2 \times 3 \times 2^2} + \frac{5}{3 \times 4 \times 2^3} + \cdots + \frac{k+2}{k(k+1) \times 2^k}$ $= 1 - \frac{1}{(k+1) \times 2^k}$ 3. Prove statement is true for $n = k+1$ $\frac{3}{1 \times 2 \times 2} + \frac{4}{2 \times 3 \times 2^2} + \frac{5}{3 \times 4 \times 2^3} + \cdots + \frac{k+2}{k(k+1) \times 2^k}$ $+ \frac{k+1+2}{(k+1)(k+1+1) \times 2^{k+1}} = 1 - \frac{1}{(k+1+1) \times 2^{k+1}}$ $\frac{3}{1 \times 2 \times 2} + \frac{4}{2 \times 3 \times 2^2} + \frac{5}{3 \times 4 \times 2^3} + \cdots + \frac{k+2}{k(k+1) \times 2^k}$ $+ \frac{k+3}{(k+1)(k+2) \times 2^{k+1}} = 1 - \frac{1}{(k+2) \times 2^{k+1}}$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Establishes the base case

	$LHS = \frac{3}{1\times2\times2} + \frac{4}{2\times3\times2^2} + \frac{5}{3\times4\times2^3} + \cdots + \frac{k+2}{k(k+1)\times2^k} + \frac{k+3}{(k+1)(k+2)\times2^{k+1}}$ $LHS = 1 - \frac{1}{(k+1)\times2^k} + \frac{k+3}{(k+1)(k+2)\times2^{k+1}}$ $LHS = 1 - \left(\frac{2(k+2)}{(k+1)(k+2)\times2^{k+1}} - \frac{(k+3)}{(k+1)(k+2)\times2^{k+1}}\right)$ $LHS = 1 - \frac{2k+4-k-3}{(k+1)(k+2)\times2^{k+1}}$ $LHS = 1 - \frac{k+1}{(k+1)(k+2)\times2^{k+1}}$ $LHS = 1 - \frac{1}{(k+2)\times2^{k+1}}$ $LHS = RHS$ $\therefore \frac{3}{1\times2\times2} + \frac{4}{2\times3\times2^2} + \frac{5}{3\times4\times2^3} + \cdots + \frac{n+2}{n(n+1)\times2^n} = 1 - \frac{1}{(n+1)\times2^n}$	
Q13ai	$\sin(8x+5x) + \sin(8x-5x)$ $= \sin 8x\cos 5x + \cos 8x \sin 5x + \sin 8x \cos 5x - \cos 8x \sin 5x$ $= 2\sin 8x \cos 5x$	1 Mark Correct solution
Q13aii	$\int \sin 8x \cos 5x\, dx$ $= \frac{1}{2}\int (\sin(8x+5x) + \sin(8x-5x))\, dx$ $= \frac{1}{2}\int (\sin 13x + \sin 3x)\, dx$ $= \frac{1}{2}\times\left(-\frac{1}{13}\cos 13x - \frac{1}{3}\cos 3x\right) + C$ $= -\frac{1}{26}\cos 13x - \frac{1}{6}\cos 3x + C$	2 Marks Correct solution 1 Mark Obtains $\frac{1}{2}\int (\sin 13x + \sin 3x)\, dx$
Q13b	$V = \pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{2}(1+\sin x)\right)^2 dx$ $V = \frac{\pi}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + 2\sin x + \sin^2 x) dx$ $V = \frac{\pi}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(1 + 2\sin x + \frac{1}{2}(1-\cos 2x)\right) dx$ $V = \frac{\pi}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{3}{2} + 2\sin x - \frac{1}{2}\cos 2x\right) dx$ $V = \frac{\pi}{4}\left[\frac{3}{2}x - 2\cos x - \frac{1}{4}\sin 2x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ $V = \frac{\pi}{4}\left[\left(\frac{3}{2}\times\frac{\pi}{2} - 2\cos\frac{\pi}{2} - \frac{1}{4}\sin\left(2\times\frac{\pi}{2}\right)\right) - \left(\frac{3}{2}\times-\frac{\pi}{2} - 2\cos\left(-\frac{\pi}{2}\right) - \frac{1}{4}\sin\left(2\times-\frac{\pi}{2}\right)\right)\right]$ $V = \frac{\pi}{4}\left[\left(\frac{3\pi}{4} - 0 - 0\right) - \left(-\frac{3\pi}{4} - 0\right)\right]$ $V = \frac{3\pi^2}{8}\ units^3$	3 Marks Correct solution 2 Marks Correct anti-derivative 1 Mark Finds $\frac{\pi}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + 2\sin x + \sin^2 x) dx$

Q13ci	$\overrightarrow{XM}=\overrightarrow{XB}+\overrightarrow{BM}$ $\overrightarrow{XM}=\frac{3}{4}\overrightarrow{AB}+\frac{1}{2}\overrightarrow{BD}$ $\overrightarrow{XM}=\frac{3}{4}\overrightarrow{DC}+\frac{1}{2}(\overrightarrow{BA}+\overrightarrow{AD})$ $\overrightarrow{XM}=\frac{3}{4}(8\boldsymbol{a}+4\boldsymbol{b})+\frac{1}{2}(-8\boldsymbol{a}-4\boldsymbol{b}-12\boldsymbol{a}-20\boldsymbol{b})$ $\overrightarrow{XM}=(6\boldsymbol{a}+3\boldsymbol{b})+(-10\boldsymbol{a}-12\boldsymbol{b})$ $\overrightarrow{XM}=-4\boldsymbol{a}-9\boldsymbol{b}$	2 Marks Correct solution 1 Mark Finds $\overrightarrow{XB}$ or $\overrightarrow{BM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$
Q13cii	$\overrightarrow{MY}=\overrightarrow{MB}+\overrightarrow{BY}$ $\overrightarrow{MY}=\overrightarrow{MB}+\overrightarrow{BC}+\overrightarrow{CY}$ $\overrightarrow{MY}=-\overrightarrow{BM}-\overrightarrow{CB}-\frac{1}{2}\overrightarrow{DA}$ $\overrightarrow{MY}=(10\boldsymbol{a}+12\boldsymbol{b})-(12\boldsymbol{a}+20\boldsymbol{b})-\frac{1}{2}(12\boldsymbol{a}+20\boldsymbol{b})$ $\overrightarrow{MY}=10\boldsymbol{a}+12\boldsymbol{b}-12\boldsymbol{a}-20\boldsymbol{b}-6\boldsymbol{a}-10\boldsymbol{b}$ $\overrightarrow{MY}=-8\boldsymbol{a}-18\boldsymbol{b}$ $\overrightarrow{XY}=\overrightarrow{XB}+\overrightarrow{BY}$ $\overrightarrow{XY}=6\boldsymbol{a}+3\boldsymbol{b}-12\boldsymbol{a}-20\boldsymbol{b}-6\boldsymbol{a}-10\boldsymbol{b}$ $\overrightarrow{XY}=-12\boldsymbol{a}-27\boldsymbol{b}$ $\overrightarrow{XY}=3\overrightarrow{XM}=\frac{3}{2}\overrightarrow{MY}$ Since these vectors are scalar multiples of each other and M is a common point, therefore X, M and Y are collinear. $k=\frac{1}{2}$	3 Marks Correct solution 2 Marks Finds k or proves collinear 1 Mark Finds $\overrightarrow{MY}$ or $\overrightarrow{XY}$
Q13di	$\frac{x+2}{4-x^2}$ $=\frac{x+2}{(2+x)(2-x)}$ $=\frac{1}{2-x}$	1 Mark Correct solution
Q13dii	$2(4-x^2)\frac{dy}{dx}=y(x+2)$ $\int \frac{2}{y}dy=\int \frac{x+2}{4-x^2}dx$ $\int \frac{2}{y}dy=\int \frac{1}{2-x}dx$ $2\ln\|y\|=-\ln\|2-x\|+C_1$ $\ln\|y\|=-\frac{1}{2}\ln\|2-x\|+C_2$ $\ln\|y\|=\ln\frac{1}{\sqrt{2-x}}+C_2$ $y=Ae^{\ln\frac{1}{\sqrt{2-x}}}$ $y=\frac{A}{\sqrt{2-x}}$ $x=1, y=2$ $2=\frac{A}{\sqrt{2-1}}$ $A=2$ $\therefore y=\frac{2}{\sqrt{2-x}}$	3 Marks Correct solution 2 Marks Obtains correct primitive 1 Mark Separates the variables in the differential equation, or equivalent merit

Q14a		3 Marks Correct solution 2 Marks Correct graph with most key features shown 1 Mark Identifies some features and one correct branch
Q14bi	$\frac{dN}{dt} = kN\left(1 - \frac{N}{5000}\right)$ $500 = k \times 1000 \times \left(1 - \frac{1000}{5000}\right)$ $500 = k \times 800$ $k = \frac{5}{8} = 0.625$	1 Mark Correct solution
Q14bii	$\frac{1}{N} + \frac{1}{5000 - N}$ $= \frac{5000 - N + N}{N(5000 - N)}$ $= \frac{5000}{N(5000 - N)}$	1 Mark Correct solution
Q14biii	$\frac{dN}{dt} = 0.625N\left(1 - \frac{N}{5000}\right)$ $\frac{dN}{dt} = 0.625N\left(\frac{5000 - N}{5000}\right)$ $\int \frac{5000}{N(5000 - N)} dN = \int 0.625 dt$ $\int \left(\frac{1}{N} + \frac{1}{5000 - N}\right) dN = \int 0.625 dt$ $\ln\|N\| - \ln\|5000 - N\| = 0.625t + C$ $\frac{N}{5000 - N} = Ae^{0.625t}$ $t = 0, N = 200$ $\frac{200}{5000 - 200} = Ae^0$ $A = \frac{1}{24}$ $\frac{N}{5000 - N} = \frac{1}{24}e^{0.625t}$ $24N = e^{0.625t}(5000 - N)$ $24N + e^{0.625t}N = 5000e^{0.625t}$ $N(24 + e^{0.625t}) = 5000e^{0.625t}$ $N = \frac{5000e^{0.625t}}{24 + e^{0.625t}}$ $t = 6$ $N = \frac{5000e^{0.625\times6}}{24 + e^{0.625\times6}}$ $N = 3196.06 \ldots$ $N = 3196$ The number of people in the arena after 6 minutes is approximately 3196.	4 Marks Correct solution 3 Marks Makes significant progress and rearranging the equation to find N 2 Marks Integrate both sides correctly 1 Mark Separating the variables in the differential equation

Q14biv	$90\% \times 5000 = 4500$ $\frac{4500}{5000-4500} = \frac{1}{24}e^{0.625t}$ $9 = \frac{1}{24}e^{0.625t}$ $9 \times 24 = e^{0.625t}$ $\ln 216 = 0.625t$ $t = \frac{\ln 216}{0.625}$ $t = 8°36'1.6''$ $t = 8\ min\ 36\ s$ (nearest second)	1 Mark Correct solution
Q14ci	Let $\sin^{-1} x = \alpha, \cos^{-1} x = \beta$ $\sin\alpha = \frac{x}{1}, \cos\beta = \frac{x}{1}$  $\sin(\sin^{-1} x - \cos^{-1} x)$ $= \sin(\alpha - \beta)$ $= \sin\alpha\cos\beta - \sin\beta\cos\alpha$ $= \frac{x}{1} \times \frac{x}{1} - \frac{\sqrt{1-x^2}}{1} \times \frac{\sqrt{1-x^2}}{1}$ $= x^2 - (1-x^2)$ $= 2x^2 - 1$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Identifies $\sin\beta = \cos\alpha$ $= \sqrt{1-x^2}$
Q14cii	$\sin^{-1} x - \cos^{-1} x = \sin^{-1}(2-x)$ $\sin(\sin^{-1} x - \cos^{-1} x) = 2 - x$ $2x^2 - 1 = 2 - x$ $2x^2 + x - 3 = 0$ $2x^2 - 2x + 3x - 3 = 0$ $2x(x-1) + 3(x-1) = 0$ $(x-1)(2x+3) = 0$ $x = 1, x = -\frac{3}{2}$ $0 \le x \le \frac{\pi}{2}$ $\therefore x = 1$ is the only solution.	2 Marks Correct solution 1 Mark Deduce $2x^2 - 1 = 2 - x$ and find both x values